Course Information

1 Who's Who in ESE 530 . . .

Instructor: Santosh S. Venkatesh
Contact: venkatesh@seas.upenn.edu
Office Hours: Mondays 3:00 – 4:30 p.m., Wednesdays 4:30 – 6:00 p.m.

TAs: Bruce Lee, Anshul Tripathi, Xiaohan Zhang
Contact: {brucele,anshult,zxiaohan}@seas.upenn.edu
Office Hours: Mondays, Tuesdays, Wednesdays 6:30 – 8:00 p.m.

Lectures: Tuesdays and Thursdays 1:45 – 3:15 p.m.

Recitations: Fridays 5:30 – 7:00 p.m.
Occasional. Will be announced on Piazza.

2 Zoom Links

The Covid–19 emergency has sadly resulted in some students being unable to travel to campus. Lectures are live but recorded on Zoom. The links are available on the course Canvas site.

3 . . . and What's What in ESE 530

This rapidly moving course provides a formal development of the mathematical foundations of the theory of probability. The course is suitable for students with a solid mathematics preparation who are seeking a rigorous graduate-level exposure to probabilistic ideas and principles with applications in diverse settings. We begin with an exploration of the fundamental concept of statistical independence, connect it via a magical 16th century trigonometric identity of Viète to the theory of numbers and, via this correspondence, discover the fundamental limit laws in probability: the venerable weak law of large numbers of Chebyshev, Borel's strong law of large numbers, and, finally, the theorem of de Moivre and Laplace which led over almost three centuries of discovery to the *éminence grise* of the theory of probability, the central limit theorem. These results already allow us to put the elementary theory seen in an undergraduate class in a peculiar and powerful new light: for instance, we will already have enough in our quiver to discuss the absurd efficacy of the ubiquitous opinion poll and why the new drug approval system works.

The rest of the course builds upon this framework: beginning with a quick review of the elementary or naïve theory of chance we move on to the formal description of probability spaces as measure spaces, construct measures, and develop a formal theory of integration, the Lebesgue integral (to replace the earlier Riemannian theory which proves to be insufficient for our purposes), before returning to the limit laws in full generality and a panoply of rich applications. We shall encounter select topics from the following list: sigma algebras; distribution functions; measure and outer measure; mathematical expectation and the Lebesgue theory; convolutions; operator methods; Laplace transforms and characteristic functions; selection principles; conditional expectation; the inequalities of Jansson, Cauchy–Schwarz, Hölder, and Minkowski; the Banach space L^p and the Hilbert space L^2; convergence in L^p; the inequalities of Chebyshev, Markov, and Chernoff; Talagrand's inequalities and concentration of measure; convergence in probability and almost surely; the law of large numbers, the law of the iterated logarithm; inclusion–exclusion, Poisson approximation, Janson's inequality, and the Stein–Chen method; the Poisson process; renewal theory; distributional convergence and the central limit theorem; Gaussian processes and Brownian motion.

A bald listing of topics does not do justice to the subject and if you find your eyes glazing over on reading the previous paragraph I don't blame you. But these dry names conceal beauty and elegance. I will be presenting the material in its lush and glorious historical context, the mathematical theory buttressed and made vivid by rich and beautiful applications drawn from the world around us. The student will see surprises in election-day counting of ballots, a historical wager the sun will rise tomorrow, the folly of gambling, the sad news about lethal genes, the curiously persistent illusion of the hot hand in sports, the unreasonable efficacy of polls and its implications to medical testing, and a host of other beguiling settings.

4 Prerequisites, contact points

This course is intended as a rigorous first graduate course in probability theory for students intending to do theoretical mathematical research in their doctoral work. (This course is *not* intended generally for M.S. students—ENM 503 or ESE 542 are the appropriate courses at the Masters level.) *Students are expected to have real mastery of basic calculus and linear algebra (as covered in the first two years of a typical undergraduate mathematics or engineering curriculum) as well as a solid foundation in undergraduate probability.* Undergraduates are warned that the course is *very* mathematical in nature with an emphasis on rigour; upperclassmen who wish to take the course will need to see the instructor for permission to register.

LECTURES

Lectures will be held in Towne 311 on Tuesdays and Thursdays from 1:45 pm to 3:15 p.m. These lectures are also recorded and streamed through Zoom for the benefit of our students who are compelled by circumstance to take the course remotely. The Zoom link is available in the course homepage on Canvas. The link is passcode protected—none of us wish to be Zoom-bombed. Please protect the link and passcode and do not pass it on to anyone else.

RECITATIONS

We have weekly recitations scheduled on Fridays from 5:30 pm to 7:00 pm. There is a separate Zoom link for recitations also available in the course homepage on Canvas. We are planning on conducting approximately ten recitations scattered through the semester: recitations will be announced on Piazza on the weeks in which they will be held. The recitations will be conducted by the teaching staff and will cover ancillary background mathematical material. I encourage you to take advantage of them; they will be your best avenue to cement understanding and expertise and fill in lacunae in your mathematical background.

ONLINE DISCUSSION FORUM ON PIAZZA

The class roster has been uploaded to Piazza from Canvas but if you use a non-SEAS email you may have to manually sign in: click on the drop down menu on the top left corner of your screen, select "+ Add Another Class" and search for ESE-530 (or type piazza.com/upenn/fall2021/ese530 into your browser's address field) to sign up for the class. The site will be monitored daily. Use the site to post questions and requests for clarification. You can ask for pointers if you get stuck on a problem but *do not post explicit solutions to problems.*

Post all class-related questions on Piazza. If you have a private concern not for public viewing you may also post messages labelled "Private" on Piazza to be viewed by the Teaching Staff only. Just toggle the appropriate link in the post. You can also contact us by email but for most communication the discussion board on Piazza will be the preferred medium.

Between the scheduled lecture time, extensive instructor and TA office hours, recitations, and the Piazza board you will have flexible access to our teaching staff. If at some point in the course you would like to set up an individual meeting with me or the staff, post a private message on Piazza and we will be glad to schedule a meeting.

5 Textbook, References

References to sections, chapters, and problems in the reading assignments and problem sets are from my book (ToP in short):

> S. S. Venkatesh, *The Theory of Probability: Explorations and Applications*. Cambridge, UK: Cambridge University Press, 2013.

You will need to get hold of a copy as I will be using the book extensively. I've had copies placed on reserve at Van Pelt (both electronic and hard cover). If you prefer to have your own copy you can also get hard copies on-line from amazon.com or Cambridge University Press, or locally from the Penn Bookstore (Barnes & Noble on 36th and Walnut). Electronic versions can also be purchased, usually at a significant discount.

If you have the time and energy for additional reading there are very many good references on probability; the following is a selected subset with my personal take on each of them.

> G. R. Grimmett and D. R. Stirzaker, *Probability and Random Processes*, Third Edition. Oxford, UK: Oxford University Press, 2002.

Grimmet and Stirzaker's book provides a reasonable balance between theory and application and is popular in engineering probability classes with a theoretical flavour. The writing is engaging though a student new to the subject will find the development a little terse with some of the more technical and difficult proofs omitted. Students may find a gap in difficulty between the development in the text and the many problems for solution. On the plus side, a less elaborate and detailed presentation permits the authors to cover a range of topics.

> R. Durrett, *Probability: Theory and Examples*, Fourth Edition. Cambridge, UK: Cambridge University Press, 2010.

Durrett's text takes a relentlessly measure-theoretic view right from the start and you should have a course in real analysis or measure theory under your belt before attempting it. (The book does contain a short appendix sumamrising the main measure-theoretic ideas but this is unlikely to be sufficient for someone without the mathematical background.) Be warned that this is a *very* terse book; the author provides spare proofs and expects the reader to fill in the details. Readers have reported a large number of errors in the manuscript—though matters have improved a little by the fourth edition—and these are likely to cause a new reader quite some bother. But, if you have the background and are wiling to fight through the details this book provides a useful complement to the course text.

There are a wealth of other books providing different perspectives and approaches. Of these, Feller's volumes are immortal and should be on the shelf of any serious student of probability.

> W. Feller, *An Introduction to Probability Theory and Its Applications, Volumes 1 and 2*. New York, NY: John Wiley.

Volume 1 deals with discrete probability and is quite remarkable still, a half-century after publication. Volume 2 is much harder—and more idiosyncratic—but still delightful.

A standard reference with a more engineering flavour is

> A. Papoulis and S. U. Pillai, *Probability, Random Variables, and Stochastic Processes*, Fourth Edition. New York, NY: McGraw–Hill, 2002.

This book was a staple in engineering probability classes for many years. It contains a grab bag of results (without attending proofs).

The book that put the theory on a rigorous footing is the classic

> A. N. Kolmogorov, *Foundations of Probability*. New York, NY: Chelsea, 1956.

Elegant, compact, and from our founding father.

6 Grading

The course grade will be based entirely on assigned homework; there are no exams or quizzes. I am currently planning on assigning eleven problem sets scattered fairly evenly through the semester but I may adjust the number depending on how rapidly we are progressing through the material.

For Ph.D. students for whom this course doubles as a qualifier, we are going to be evaluating student performances holistically based on their entire course performance in lieu of a single Ph.D. Qualifier exam for the course. This gives us an opportunity to learn this beautiful subject without fear.

In every calamity there is a silver lining.

7 Homework

A serious student of a subject is not an idle spectator to a variety show but learns best by active involvement. This is particularly true of mathematical subjects. I will accordingly be assigning problem sets regularly on a weekly to bi-weekly basis depending on class progression. Very few of these problems are of a cookbook nature; in some cases they build on the developments in the main body of the text and in others explore significant new areas. The weight given to homework in grading is in recognition of its central importance in building understanding.

We will be progressing at a ferocious pace and you will find that part of the challenge in managing your time is that I will simultaneously be giving you the responsibility of exploring beguiling digressions and background material to support and extend what we cover in class while also assigning homework problems to help cement your understanding. This course demands—and rewards—consistent and continuous effort; sporadic work and sudden spurts of activity, on the other hand, will lead to frustration. So plan accordingly and commit your time wisely and regularly. On the plus side, this is a truly beautiful subject and your reward will be in the richness of the understanding that you will develop.

Collaboration and Reference Policy

Collaboration on homework in study groups is encouraged. While such collaboration in the sense of discussions is allowed, *students must write up the final solutions of the homework problems alone and not simply copy the material from another source.* All collaborators *must* be clearly and explicitly acknowledged on the first page of the submitted homework. Acknowledged collaboration will have no effect on the received grade but is demanded by intellectual probity. Unacknowledged collaboration or plagiarism is theft and the consequences are grave; do yourself a favour and stay on the side of the angels.

Reference to someone else's solutions, current or past, or solutions to problem sets that I have distributed in previous incarnations of this course are expressly forbidden, as are explicit solutions for problems found on the web or in a reference. Anti-intellectual behaviour such as plagiarism will be regarded most severely. This has no place in a community of scholars and students are expected, nay constrained, by the honour system to comport themselves with the utmost integrity.

Homework submissions on Gradescope

Create an account (if needed) and log in at www.gradescope.com. Then, click "Enroll in Course" on the lower right hand corner of the screen. Our six-character course entry code is 5VPWKW.

All homework should be uploaded electronically to Gradescope. Problem sets will typically be assigned on Thursdays and will be due *one week from assignment* (with the occasional rare exception proving the rule). Please ensure that your homework is turned in on time as ***late submissions will generally not be accepted***. I do understand that remote learning places particular challenges on individuals. Reach out to me or our teaching staff if you have particular circumstances which force a delay in submission and we will work with you to find a reasonable solution: cases where we will grant extensions include family emergencies, time zone mismatches, and technological breakdowns (for example, storm outages); cases where we will generally *not* grant exceptions include crowded work weeks, other deadlines and projects due, and exams in other classes.

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